HOW TO PRICE & HEDGE VARIABLE ANNUITIES WITH UNHEDGEABLE RISK

PRMIA
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Topics

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1. Risk: the hedgeable, the diversifiable and the unbearable
2. The decomposition of surrender risk
3. Pricing & valuation
4. Transformation of policies
5. Case study: target volatility fund derivatives

Typical GMAB / DB product (with knockout)

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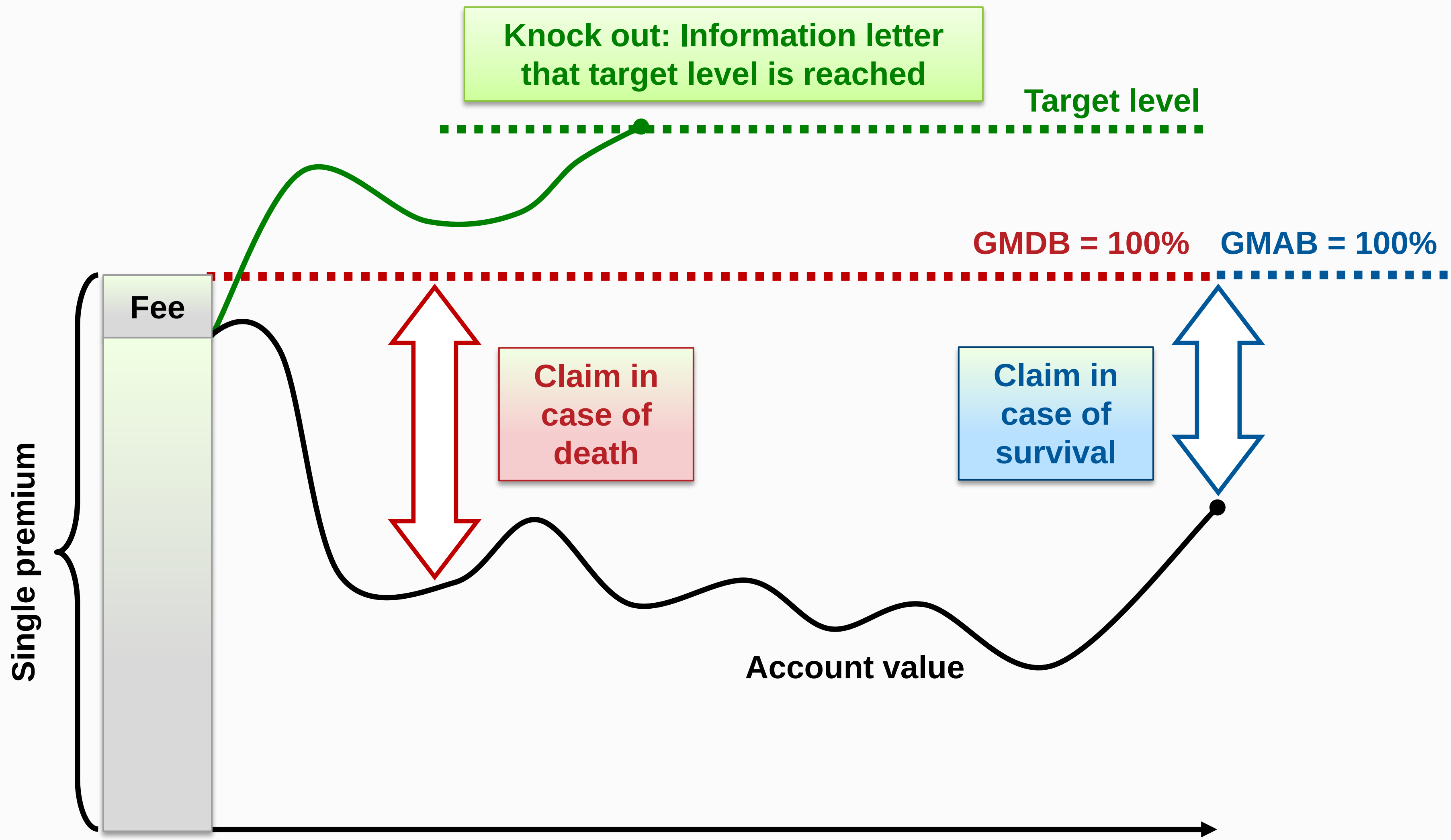

Pricing Principles & Risk Analysis

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Hedgeable:	Not hedgeable, but measurable:	Moral hazard / not well measurable:
▪ Exchanges: equity, interest rates, FX futures ▪ Vanilla OTC: equity index options, variance swaps, interest rate swaps, swaptions ▪ predictable surrender	▪ Basis risk hedge versus fund benchmarks ▪ Mortality ▪ Diversity in surrender experience unrelated to capital market ▪ Very long-term interest rates and vega	▪ Benchmark versus actual fund performance ▪ Fund switching option ▪ New business volume ▪ Surrender induced by sales network ▪ Longevity and other long-term trend risk
Market-consistent value of hedgeable component of the liability	**Cost of capital for non-hedgeable component of the liability**	**Limits, exclusions, conservative pricing paired with profit participation**

Surrender Risk
Experience Analysis

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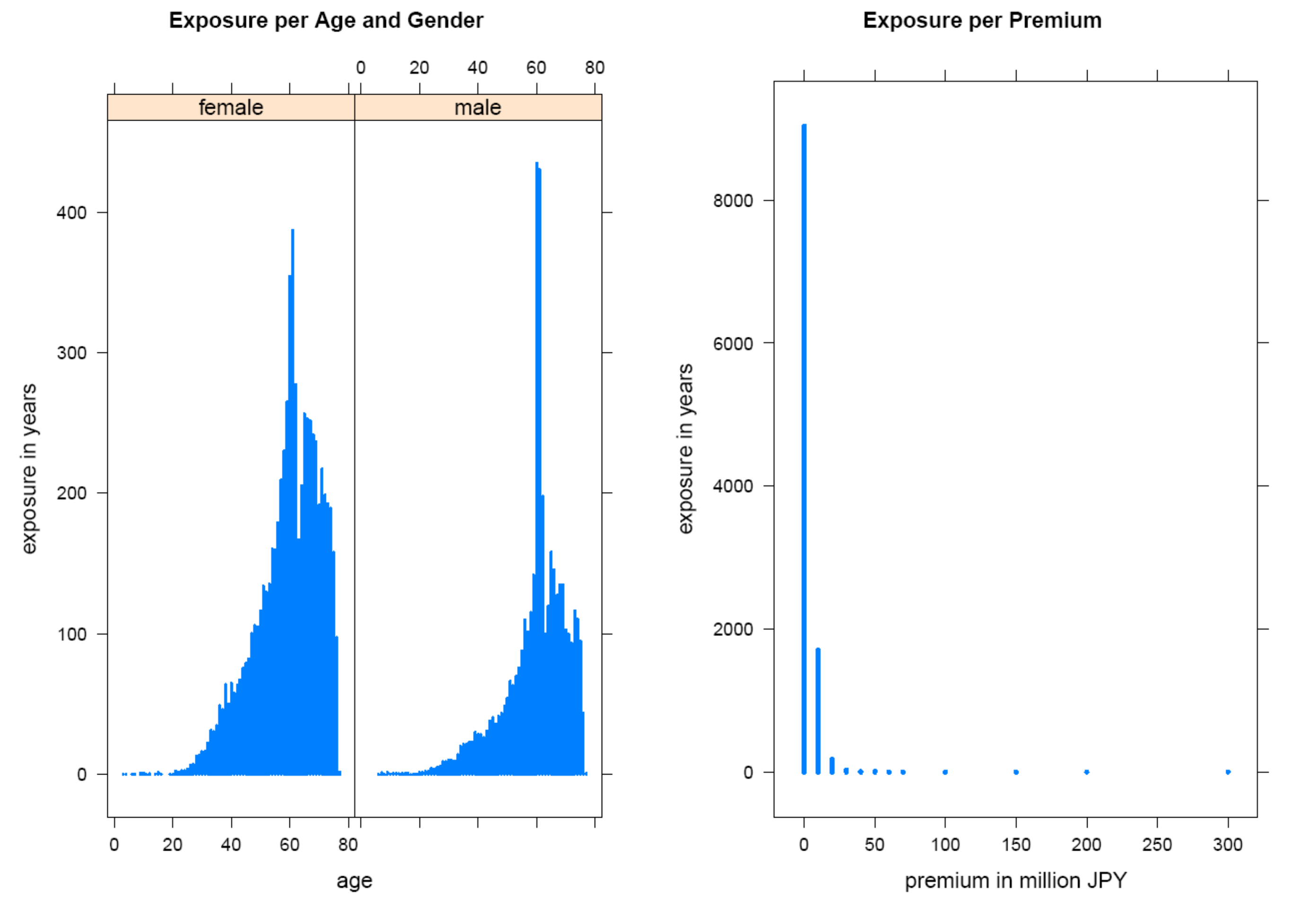
contract	number of policies	policies written
1		01.04.2008 - 03.09.2009
total	**30306**	

experience

period	exposure years	# surrenders	# deaths	# knock-out
Apr 09 – Feb 10	**20646**	**749**	**53**	**31**

Surrender Experience Analysis
Explaining Variables (1)

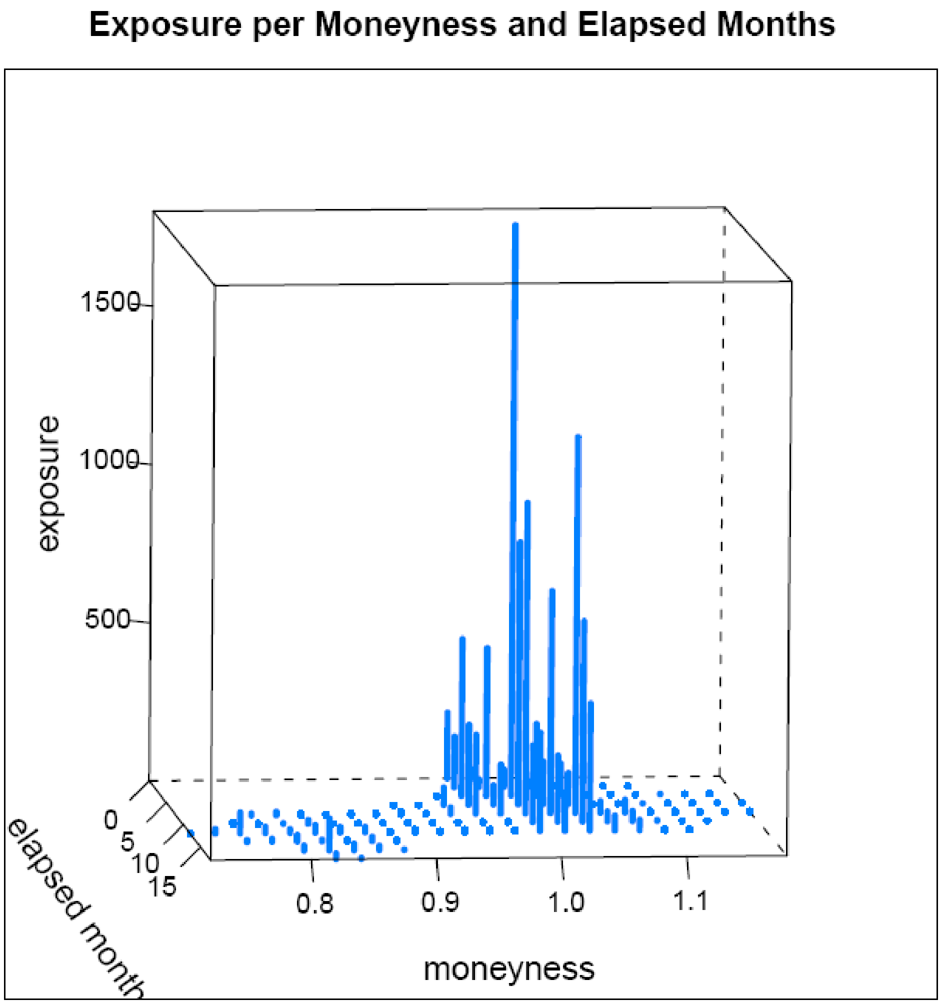
First understand the distribution of the explaining variables:



- this is a savings product favored by 60-year-olds
- distribution of the invested amount (the premium) is highly skewed

Surrender Experience Analysis
Explaining Variables (2)

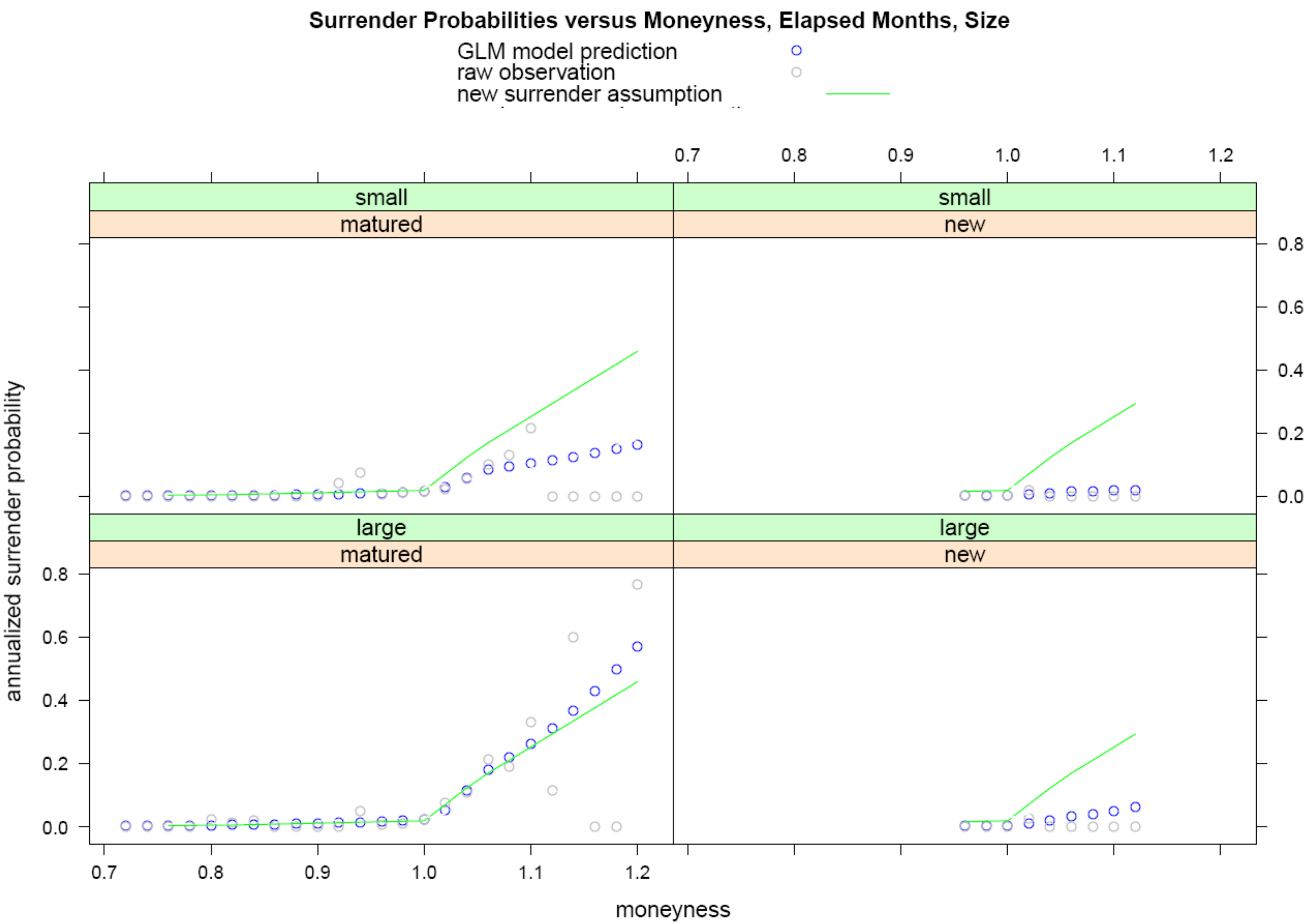
First understand the distribution of the explaining variables:



- moneyness and elapsed months are sufficiently spread
- both can be used as explaining variables

Surrender Experience Analysis
Generalized Linear Model

Use only moneyness, size and the age of the policy as explaining variables:



- as expected: we observe a limited rationality in the surrender behavior
- surrender rates are significantly lower in the first three months after inception
- surrender rates are more rational for larger sizes

Combining actuarial and market consistent modeling
The market-consistent production value

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» Decomposition of the net liability into two components:

$$L = H + R$$

» Common core in Swiss Solvency Test, Solvency II, IFRS phase II, CRO-Forum, CFO-Forum*:

The market-consistent production value of the liability is computed as

$$v(L) = m(H) + c(R)$$

where m(H) is the cost of setting up and running the hedging strategy (including upfront and expected future transaction costs) and c(R) is the cost of the risk capital required to carry the residual risk:

$$c(R) = \text{“hurdle rate”} * \text{“risk capital contribution of R”}$$

Legend

L = Liability

H = Hedgeable component

R = Residual risk

v(..) = Value of …

c(..) = Costs of …

m(...) = Market value of …

“Hurdle rate” = the return on risk capital required in MR Group

“Risk capital contribution of R” = the amount by which MR’s group risk increases due to the additional risk R

Hedging strategy determines the production value.

Pre-2003 state of non-convergence

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Banks' complete market models cannot explain:

- bid-ask spreads
- when you should hedge and when not
- how to extrapolate (e.g. term structure of interest rates and term structure of implied vol)
- how to value non-traded insurance risks

Insurers' "real world models" can lead to suboptimal behavior:

- potential to be outsmarted by banks if not market consistent
- potential to buy "toxic waste" without getting a "fair price" for it (CRO Forum: "Financial Risk Mitigation")
- potential conflict with shareholder's desire for both transparency and short-term P&L stability

Transformation of Policy Data

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Main idea goes back to Kalberer (2006):

*payoff(p,s,t) = weight(p,t) * f(g(p,t),s,t)*

p: policy parameters
s: scenario
t: date
g(p): transformed policy parameters (= parameters of a financial derivative)

Task: find appropriate f and g, such that g(p) has lower dimension than p!

Example GMDB:

GMDB = $\sum_t$ *Put(with exercise date t) * "Probability that policyholder dies at t"*

Separate the dependency on policy parameters as much as possible from the dependency on the market scenarios.
=> Approximate cash flows (not PVs!) rather model-free.

Transformation of Policy Data

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product	number of policies	number of model points in traditional approach, 5% bucket size in moneyness	number of basis products in transformation approach, 5% bucket size in moneyness
1	26306	2736	20
2	36738	2635	8
3	168	142	45
4	1099	525	15
5	776	391	10
6	235	156	8
7	46	44	20
8	222	176	12
9	1835	1020	50

Target Volatility Fund (TVF) mechanics

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A target volatility fund consists of a risky component and a virtually risk-less "control component". The realized volatility σ_t of the risky component is measured with a pre-determined formula and the weight of the risky component for the next business day is then chosen as $w_{t+1} = \sigma_{target} / \sigma_t$, based on today's volatility estimate σ_t. The two main types of volatility estimates are

- equal-weighted estimates of realized volatility and
- exponentially weighted estimates of realized volatility:

$$\sigma_t^2 = \lambda \, \sigma_{t-1}^2 + (1 - \lambda) \, 252 \, R_t^2 ,$$

where R_t is the log return of the risky component $R_t = \log(S_t/S_{t-1})$.

Base case:
- daily returns R_t,
- exponential smoothing with $\lambda = 0.975$,
- daily rebalancing, no thresholds (ignores transaction costs for the moment)

Vega Profiles

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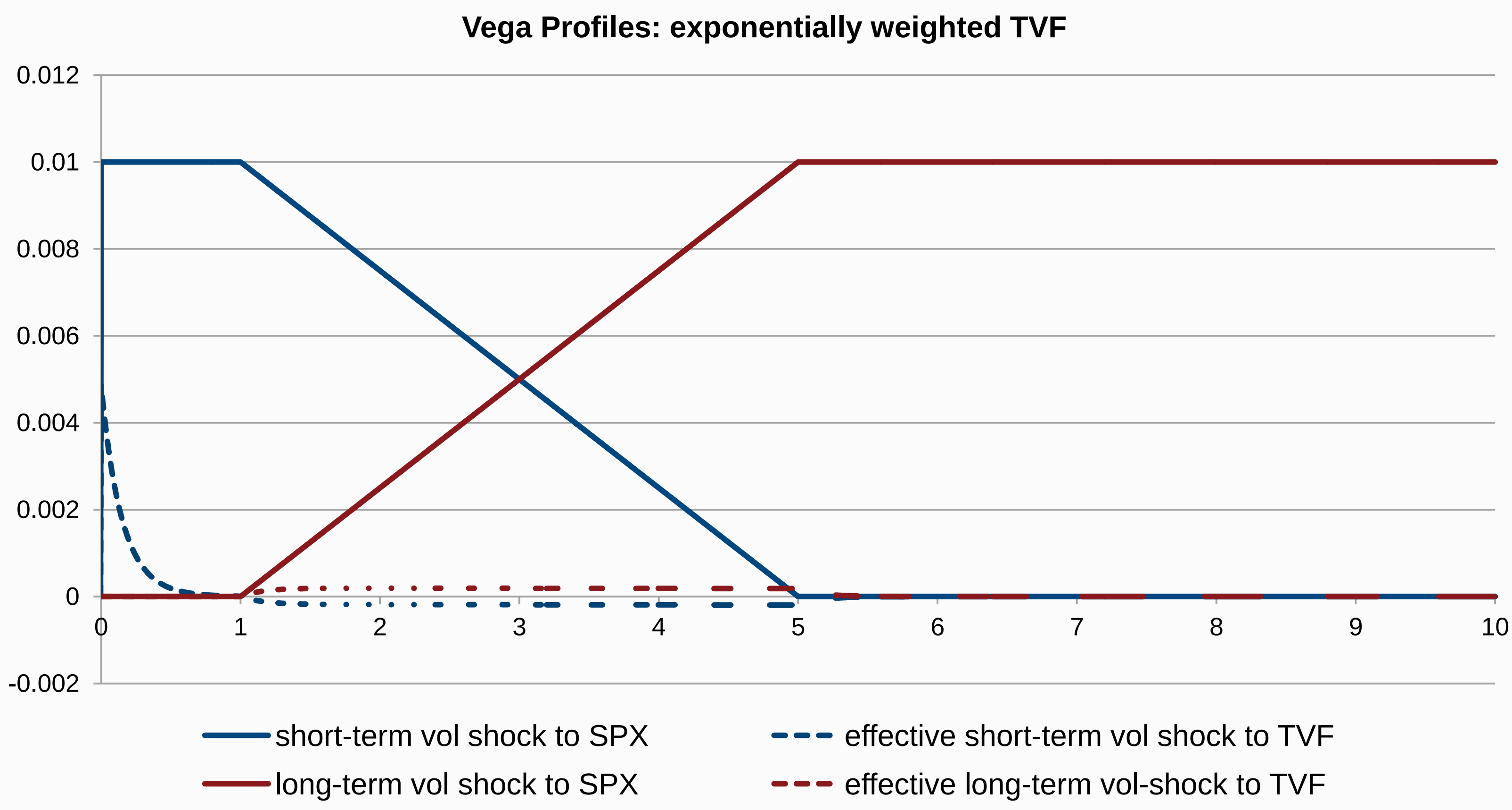

Stylized fact:
- exponential smoothing almost completely removes long-term vega
- short-term vega remains

Gamma Profiles

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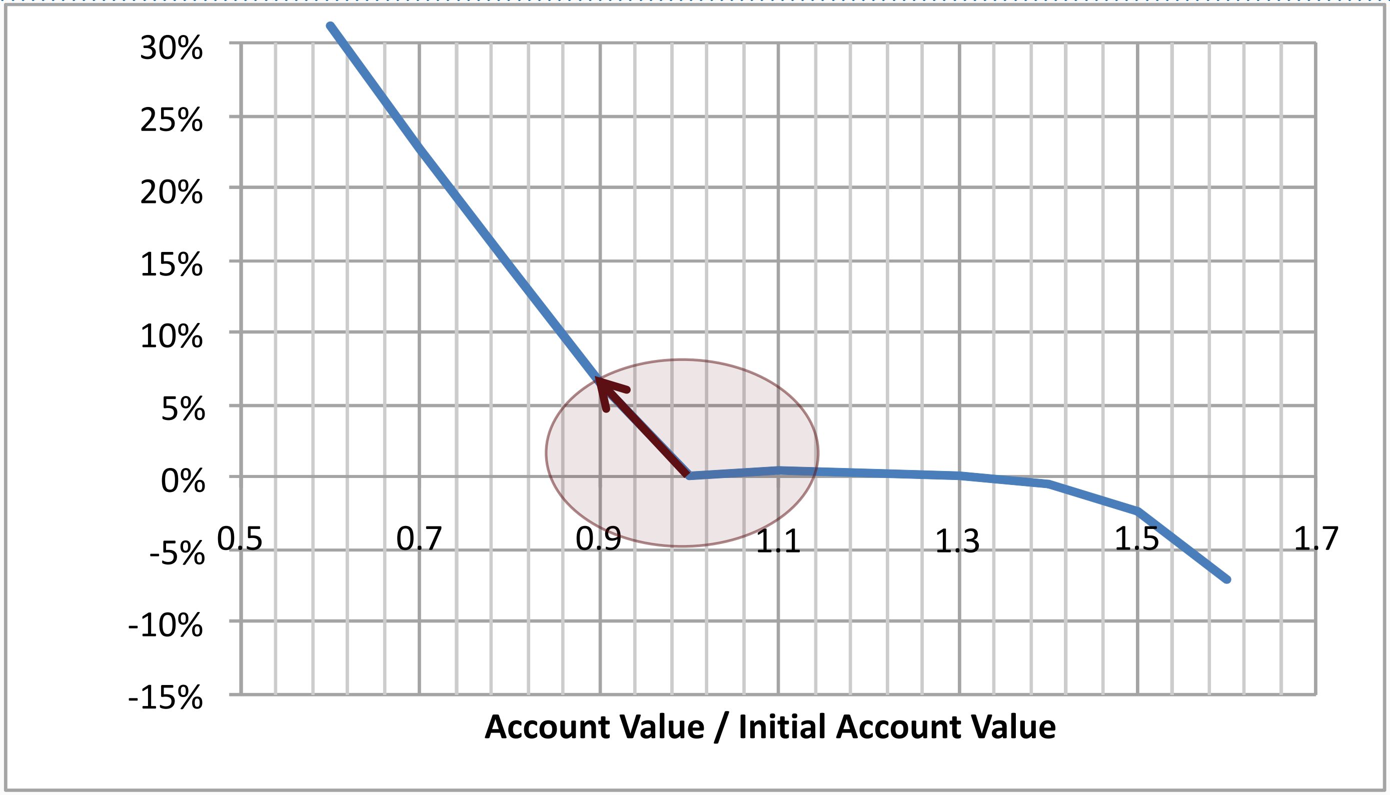
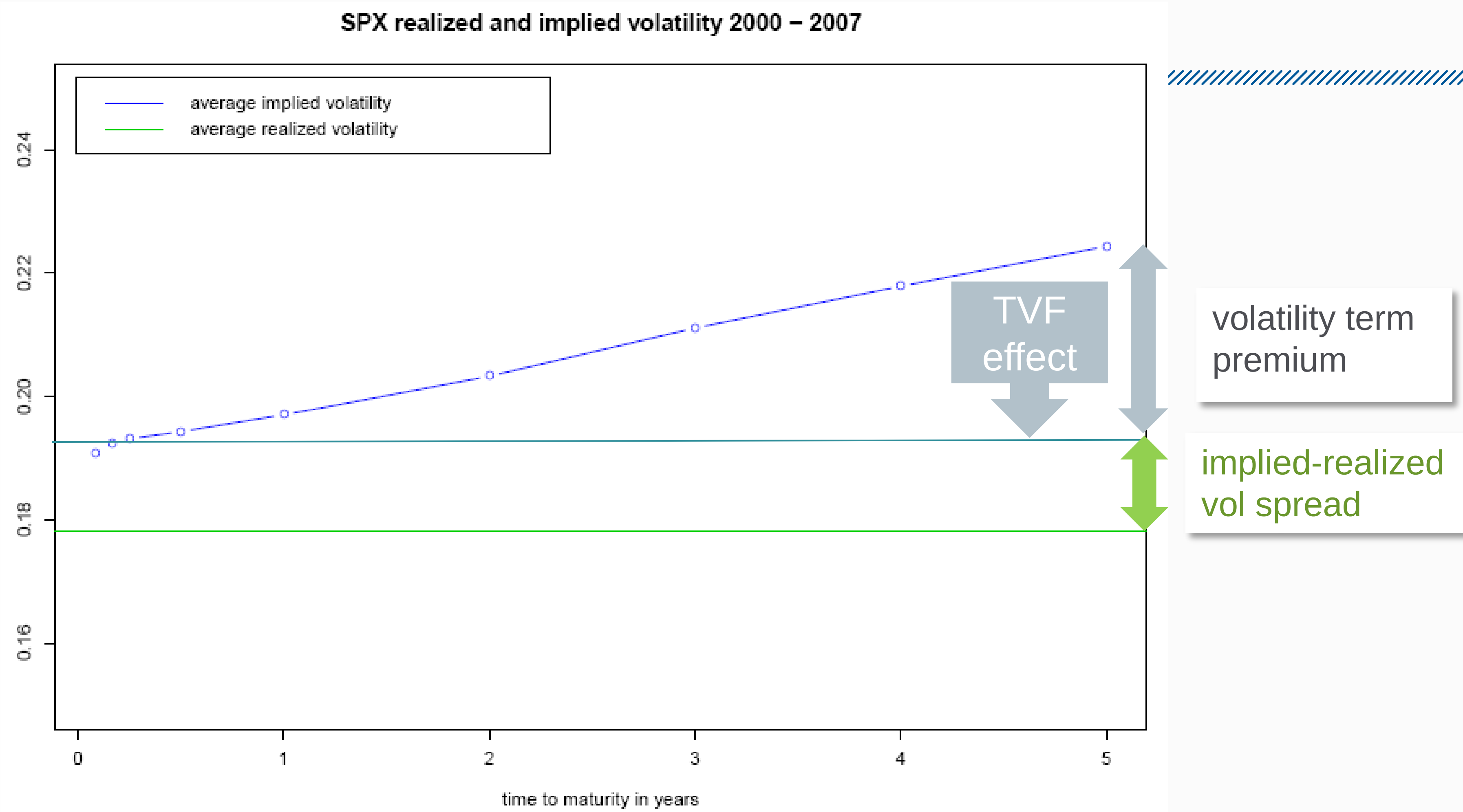


10% drop in AV causes 7% of single premium loss (if un-hedged) Delta-hedge could reduce this to 3.5%.

Stylized fact:

- non-linearity (gamma) in the PV profile versus the underlying remains almost un-effected by the TVF mechanism
- large gamma is caused by dynamic surrender, ratchets/clicks – which is not mitigated by TVF mechanism
- TVF derivatives need to be hedged by rolling over short-term (1-3 months) hedge instruments like puts or variance swaps on the TVF underlying(s).

The volatility term premium

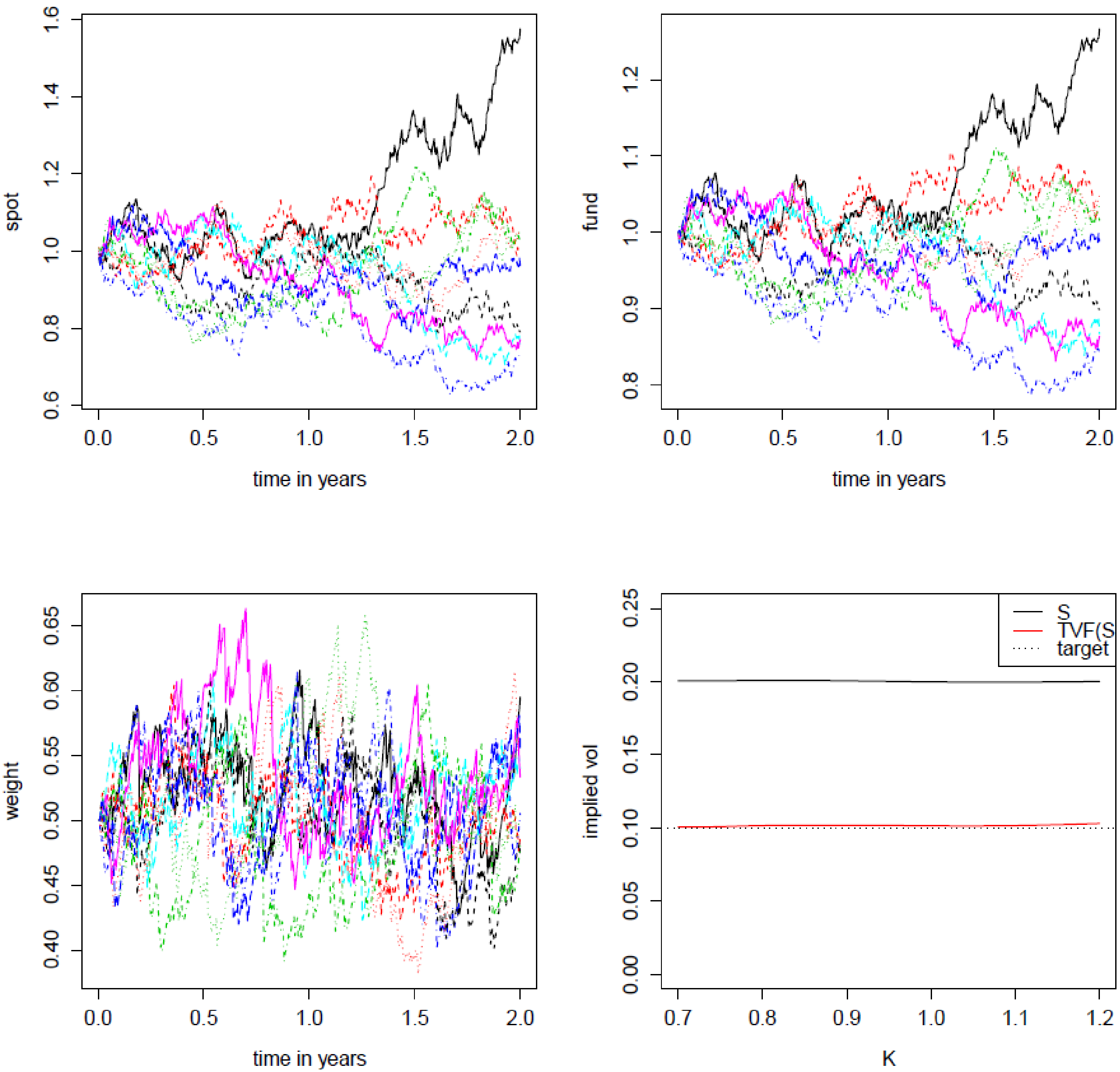


Stylized fact: The TVF mechanism

- removes the *volatility term premium* (the spread long-term implied volatility minus short-term implied volatility) but
- does not remove the short-term implied-realized volatility spread.

Put on TVF

a=c(0.2,0,0),b=c(0,0,0),lambda=c(0,0),kappa=1,v0=0.2,tv=0.1,cap=1,alpha=0.97

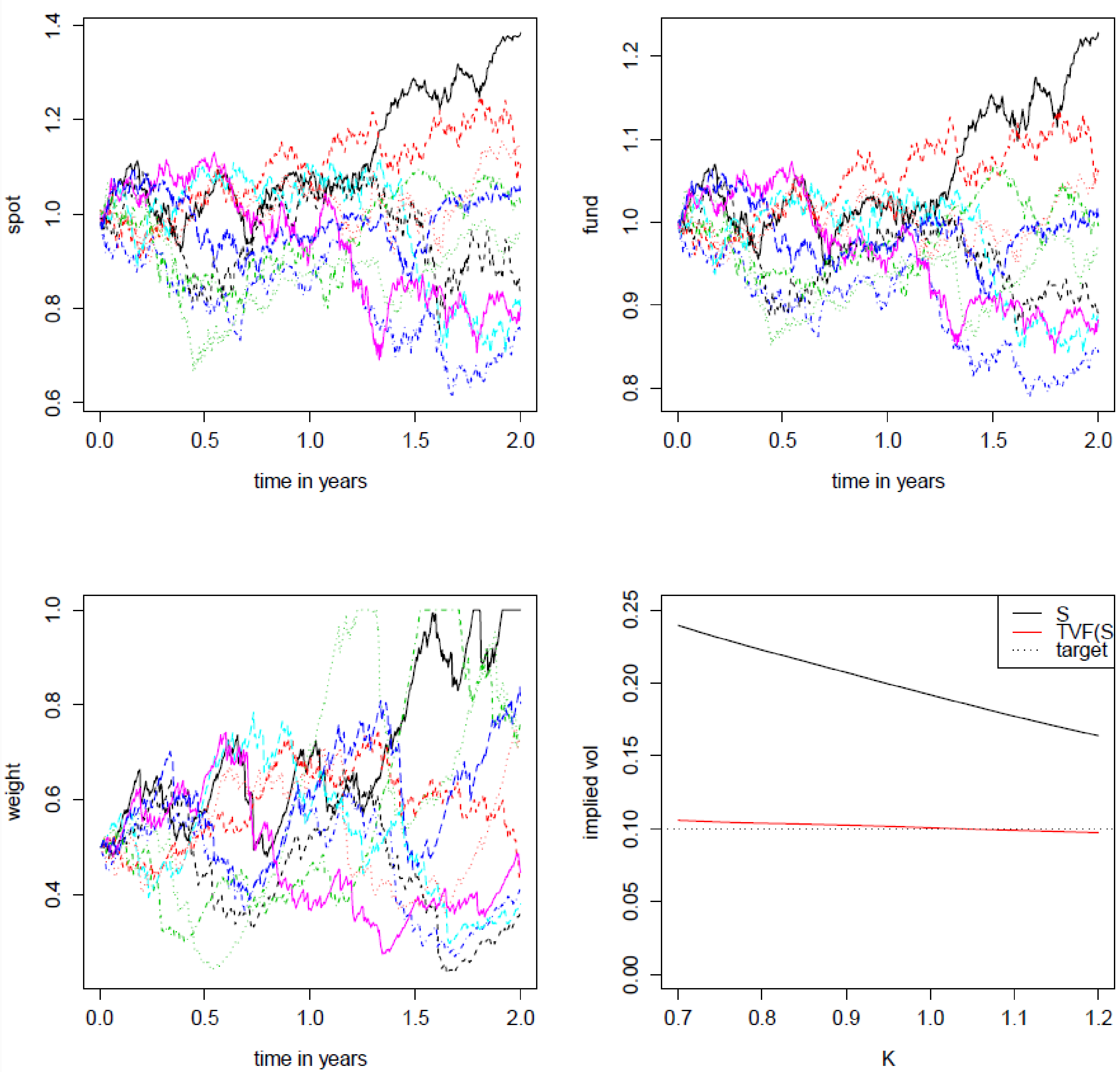


GBM:

- estimation noise leads to very slightly higher implied vol than target vol

Put on TVF

a=c(0.2,0,0),b=c(-0.8,0.02,0.02),lambda=c(50,50),kappa=2,v0=0.2,tv=0.1,cap=1,alpha=0.97

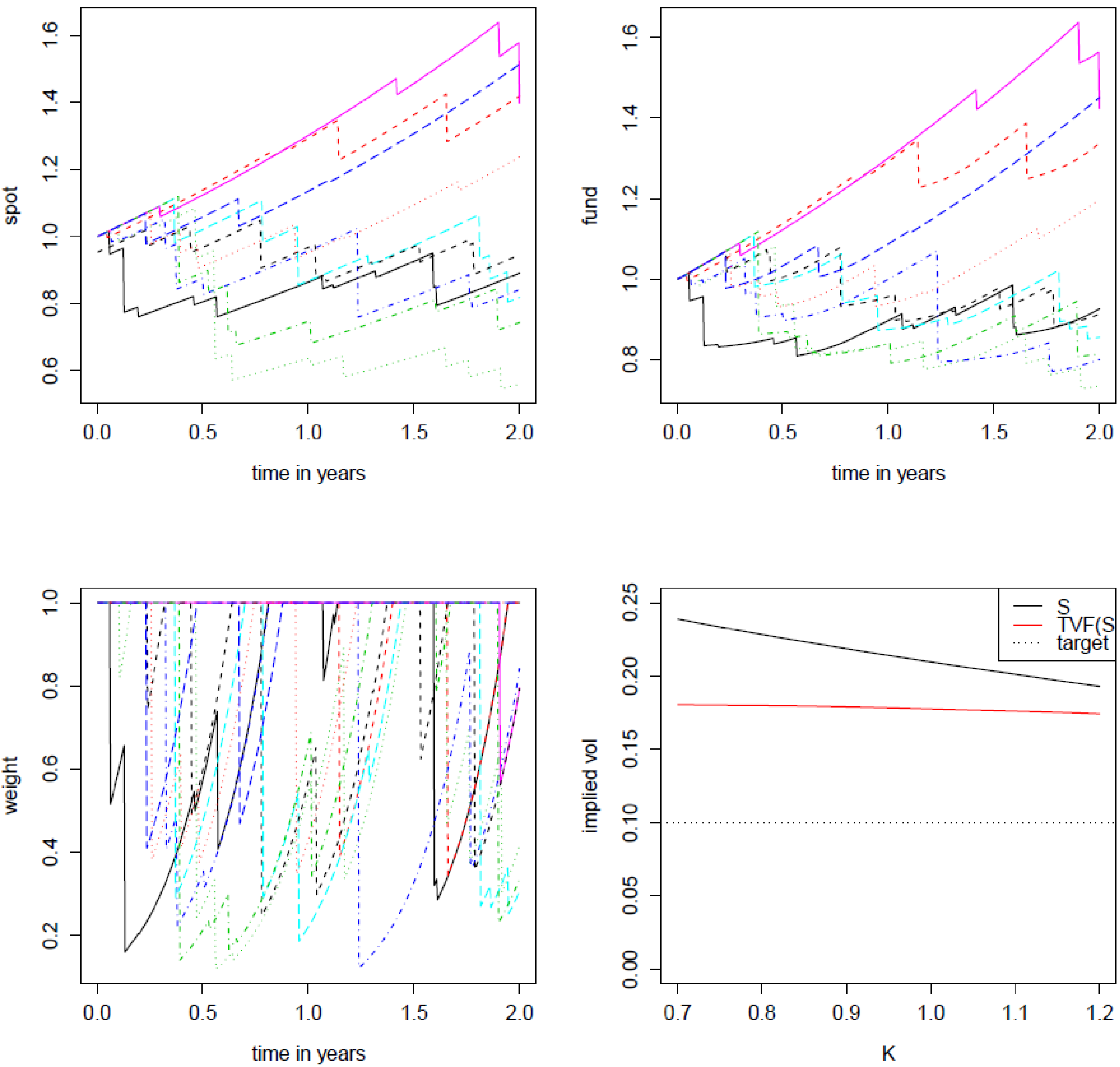


stochastic vol:
"beauty case" for TVF:

- skew removed
- pricing vol very close to target vol

Put on TVF

a=c(0,0,0.08),b=c(0,0,0),lambda=c(0,4),kappa=1,v0=0.2,tv=0.1,cap=1,alpha=0.97

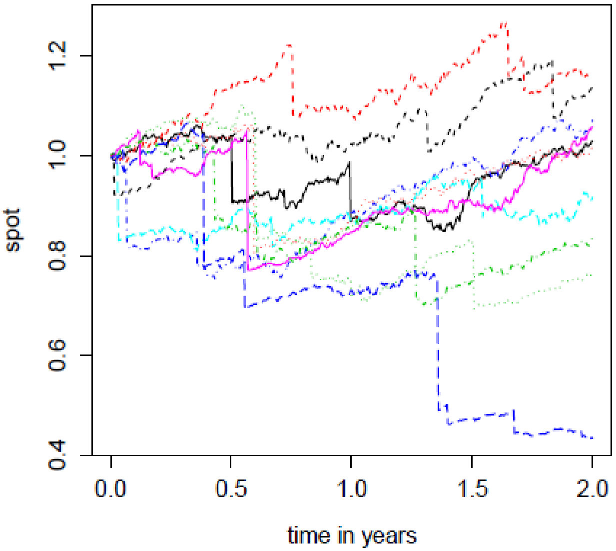


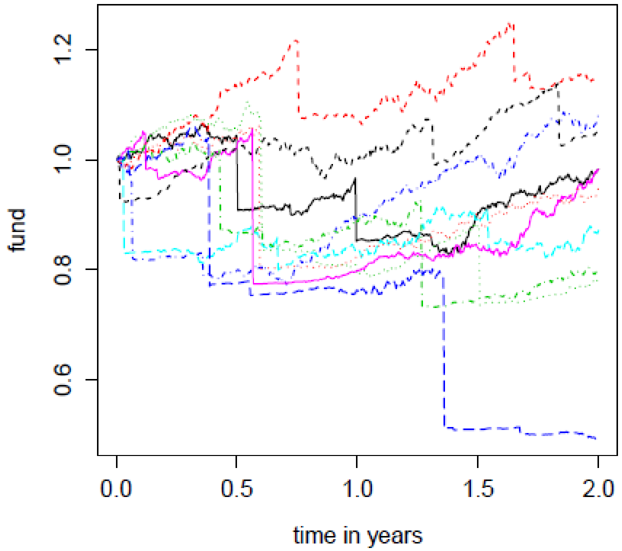
pure negative jumps:

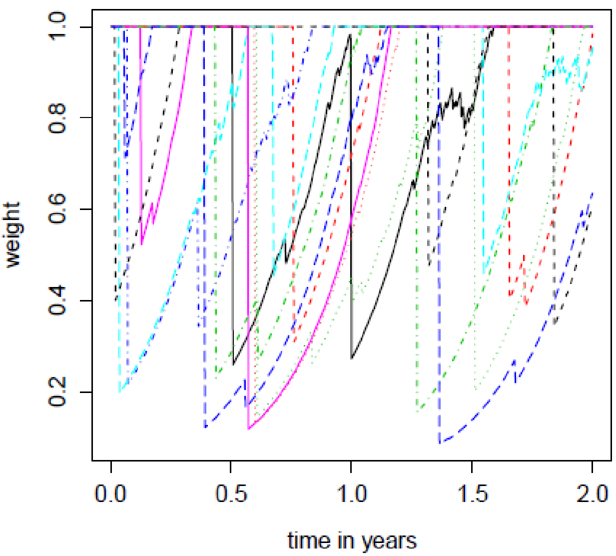
- skew removed, but
- pricing vol much larger than 120% * target vol

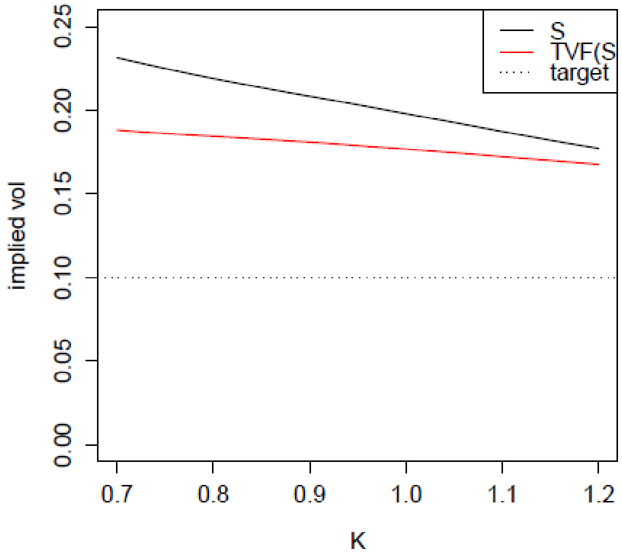
Put on TVF

a=c(0.08,0,0.1),b=c(-0.5,0,0.1),lambda=c(2,2),kappa=2,v0=0.05,tv=0.1,cap=1,alpha=0.97









still worse:

- pricing vol again much larger than 120% * target
- pricing vol mainly constrained by cap=1

Summary

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1. Explicit guarantees are good for shareholders, policyholders and regulators: increased transparency.

2. Surrender can be dangerous. Monitoring surrender experience increases hedge efficiency.

3. Valuation of liabilities and hedging are closely related. The “production value” approach to valuation aligns the valuation with the actually chosen hedge instruments. Calibrate to “deep and liquid markets” only.

4. The risk management of capital market risks embedded in life (re) contracts is non-trivial. A bank-like operation (“hedging platform”) is needed for quantification, hedge design and controlling of the risks. Economies of scale are needed to run the operation successfully. Transformation of large numbers of policies is part of the story.

5. Analysis of TVF derivatives requires explicit modeling of jumps.